ELEMENTOS DE MÁQUINAS I – CEFET-RJ – PROF. PAULO.

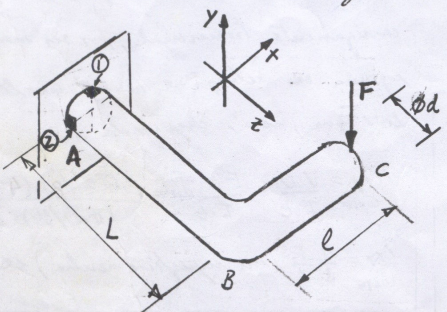
REVISÃO

Determine para a barra, de seção circular, da figura a seguir:

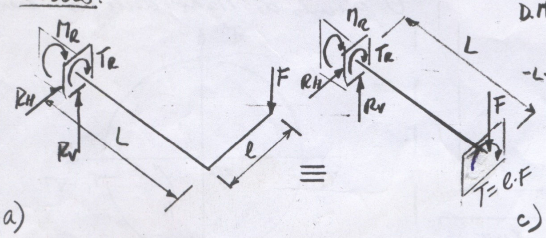
a) Reações no engaste,

b) Os diagramas de CORTANTE e de MOMENTO do trecho AB,

c) A tensão equivalente, segundo Mises, nos pontos indicados na seção engastada.



MODELO:



a)

$\overset{+}{\nearrow} \Sigma F_x = 0$

$\boxed{R_H = 0 \ N}$

$+\uparrow \Sigma F_y = 0$

$R_V - F = 0 \rightarrow \boxed{R_V = F \quad N}$

$\circlearrowleft \Sigma M_A = 0$

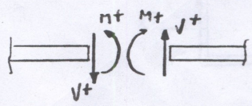
$-M_R - L \cdot F = 0 \rightarrow \boxed{M_R = -L \cdot F \quad N \cdot m}$

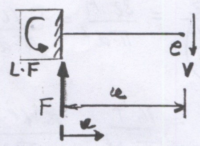
$\circlearrowleft \Sigma T_A = 0$

$-T_R - \ell \cdot F = 0 \rightarrow \boxed{T_R = -\ell \cdot F \quad N \cdot m}$

b)

Convenção



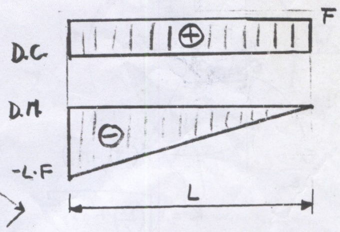


$+\uparrow \Sigma F_y = 0$

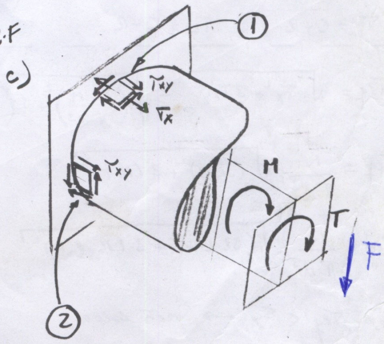
$F - V = 0 \rightarrow \boxed{V = F \quad N}$

$\circlearrowleft \Sigma M_e = 0$

$-x \cdot F + L \cdot F + M = 0 \rightarrow \boxed{M = (x - L) \cdot F \quad N \cdot m}$

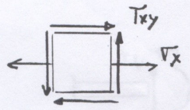


c)



c) (continuação)

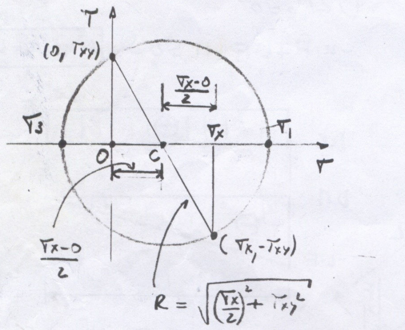
ponto 1:



$$\sigma_x = \frac{M \cdot C}{I} = \frac{M \cdot d/2}{\left(\frac{\pi \cdot d^4}{64}\right)} = \frac{32 \cdot M}{\pi \cdot d^3} = \frac{32 \cdot (l \cdot F)}{\pi \cdot d^3}$$

$$\tau_{xy_{\text{TORÇÃO}}} = \frac{T \cdot C}{J} = \frac{T \cdot d/2}{\left(\frac{\pi \cdot d^4}{32}\right)} = \frac{16 \cdot T}{\pi \cdot d^3} = \frac{16 \cdot (l \cdot F)}{\pi d^3}$$

O círculo de Mohr deste caso seria:



$\sigma_1 = C + R$ e $\sigma_2 = C - R$

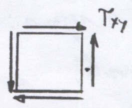
$$\sigma_{eq} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \sqrt{\left(\frac{32 \cdot M}{\pi \cdot d^3}\right)^2 + 3\left(\frac{16 \cdot T}{\pi \cdot d^3}\right)^2}$$

$$\sigma_{eq} = \frac{1}{\pi \cdot d^3} \cdot \sqrt{(32 \cdot M)^2 + 3 \cdot (16 \cdot T)^2}$$

$$\sigma_{eq} = \frac{1}{\pi \cdot d^3} \cdot \sqrt{(32 \cdot l \cdot F)^2 + 3 \cdot (16 \cdot l \cdot F)^2}$$

Se $\sigma_{eq} < S_y \rightarrow$ não escoa.

ponto 2:



$$\tau_{xy_{\text{TORÇÃO}}} = \frac{16 \cdot T}{\pi \cdot d^3} = \frac{16 \cdot (l \cdot F)}{\pi \cdot d^3}$$

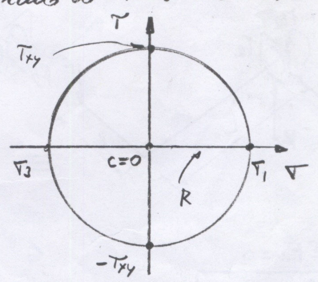
OBS:

A tensão cisalhante provocada pelo carregamento transversal, por ser muito pequena em relação a tensão cisalhante de torque, será desprezada:

$$\tau_{xy_{\text{CORTE}}} = \frac{V \cdot Q}{I \cdot t} = \frac{F \cdot A \cdot \bar{y}}{I \cdot t} = \frac{F \cdot (\pi \cdot d^2/8) \cdot (4d/6\pi)}{(\pi \cdot d^4/64) \cdot d}$$

$$\tau_{xy_{\text{CORTE}}} = \frac{4}{3}\frac{F}{A}$$ (na superfície neutra) $<< \tau_{xy_{\text{TORÇÃO}}}$

O círculo de Mohr deste caso seria:



$\sigma_1 = R$ e $\sigma_2 = -R$

$$\sigma_{eq} = \sqrt{\underset{0}{\sigma_x^2} + 3\tau_{xy}^2} = \sqrt{3 \cdot \left(\frac{16 \cdot T}{\pi d^3}\right)^2}$$

$$\sigma_{eq} = \sqrt{3} \cdot \frac{16 \cdot T}{\pi d^3}$$

$$\sigma_{eq} = \sqrt{3} \cdot \frac{16(l \cdot F)}{\pi \cdot d^3}$$

Se $\sigma_{eq} < S_y \longrightarrow$ não escoa.

OBS:

Critério de von Mises:

- Para um estado triaxial:

$$\sigma_{eq} = \frac{\sqrt{2}}{2} \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

- Para um estado biaxial (por exemplo $\sigma_2 = 0$):

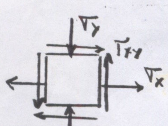
$$\sigma_{eq} = \frac{\sqrt{2}}{2} \cdot \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_3^2 - 2\sigma_1\sigma_3 + \sigma_1^2}$$

$$\sigma_{eq} = \frac{\sqrt{2}}{2} \cdot \sqrt{2 \cdot (\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2)}$$

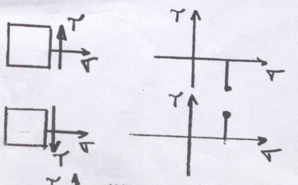
$$\therefore \sigma_{eq} = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2} \qquad (6.9)$$

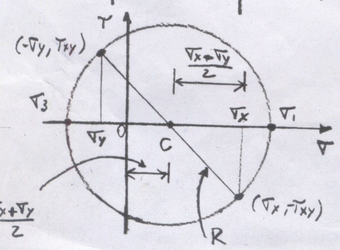
onde, σ_1, σ_2 e σ_3 são as tensões principais.

Pode-se transformar as tensões principais em tensões normais e tensões cisalhantes através da utilização do Círculo de Mohr:

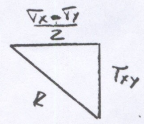


Convenção:





mas $C = \sigma_x - \left(\frac{\sigma_x - \sigma_y}{2}\right) = \frac{\sigma_x + \sigma_y}{2}$



$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\therefore R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

donde:

$$\sigma_1 = C + R = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_3 = C - R = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Ou ainda:

$$\sigma_{eq} = \sqrt{(C+R)^2 - (C+R)(C-R) + (C-R)^2}$$

$$= \sqrt{C^2 + 2CR + R^2 - C^2 + R^2 + C^2 - 2CR + R^2}$$

$$= \sqrt{C^2 + 3R^2}$$

$$= \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + 3 \cdot \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 3 \cdot \tau_{xy}^2}$$

$$= \sqrt{\frac{1}{4} \cdot (\sigma_x^2 + 2\sigma_x\sigma_y + \sigma_y^2 + 3\sigma_x^2 - 6\sigma_x\sigma_y + 3\sigma_y^2) + 3\tau_{xy}^2}$$

$$= \sqrt{\frac{1}{4} \cdot (4\sigma_x^2 - 4\sigma_x\sigma_y + 4\sigma_y^2) + 3\tau_{xy}^2}$$

$$\therefore \boxed{\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}}$$

OBS: Se uma das tensões normais for nula ($\sigma_y = 0$, por exemplo), então:

$$\boxed{\sigma_{eq} = \sqrt{\sigma_x^2 + 3 \cdot \tau_{xy}^2}} \qquad (6.11)$$